University of California, Berkeley
Physics 105 Fall 2000 Section 2 (*Strovink*)

EXAMINATION 1

Directions. Do both problems (weights are indicated). This is a closed-book closed-note exam except for one $8\frac{1}{2} \times 11$ inch sheet containing any information you wish on both sides. You are free to approach the proctor to ask questions – but he will not give hints and will be obliged to write your question and its answer on the board. Roots, circular functions, *etc.*, may be left unevaluated if you do not know them. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Box or circle your answer.

1. (50 points)
A book has its front cover facing up (the normal to the cover is along $\hat{z}$). Its sentences are parallel to the $\hat{x}$ direction, and its binding is parallel to the $\hat{y}$ direction. Consider the "body" (x, y, z) axes to be attached to the book, with their origin at its CM. Define the "space" (x', y', z') axes initially to be the same as the (x, y, z) axes; however, the (x', y', z') axes are fixed – they don't change when the book rotates.

(**a**) (10 points)
Suppose the book is rotated about its z axis by 45° counterclockwise (carrying the body axes with it). The space axes remain fixed. Write down the transformation matrix Λ_a^t such that

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \Lambda_a^t \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} .$$

(**b**) (10 points)
Instead suppose the book is rotated about its x axis by 45° counterclockwise (carrying the body axes with it). The space axes remain fixed. Write down the transformation matrix Λ_b^t such that

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \Lambda_b^t \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} .$$

(**c**) (15 points)
Instead suppose the book is first rotated as in (**a**), next rotated as in (**b**). Write down the transformation matrix Λ_c^t such that

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \Lambda_c^t \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} .$$

(**d**) (15 points)
Write down the inverse of Λ_c^t .

2. (50 points)
With respect to a fixed set of Cartesian coordinates (x, y, z), the position $\mathbf{r}(t)$ of a particle of mass m is given by

$$\mathbf{r}(t) = \hat{\mathbf{x}} x_0 + \hat{\mathbf{y}} v_0 t ,$$

where x_0 and v_0 are constants.

(**a**) (10 points)
With respect to the origin, write down the particle's moment of inertia $I(t)$.

(**b**) (15 points)
With respect to the origin, write down the magnitude and direction of the particle's *angular* velocity $\vec{\omega}(t)$.

(**c**) (10 points)
For the conditions specified in this problem, the product of $I(t)$ and $\vec{\omega}(t)$ is $\mathbf{L}$, the particle's angular momentum with respect to the origin. Write down $\mathbf{L}$. Is it a function of time t? If so, a torque must be acting on the particle – what is the source of this torque? Explain.

(**d**) (15 points)
Imagine that the particle in (**a**)-(**c**) is an element of fluid. The fluid's velocity field $\mathbf{v}(\mathbf{r})$ is given by

$$\mathbf{v}(\mathbf{r}) = \hat{\mathbf{y}} v_0 \frac{x}{x_0} ,$$

where, as above, x_0 and v_0 are constants. Can $\mathbf{v}(\mathbf{r})$ be expressed as the gradient of a scalar field $u(\mathbf{r})$, *i.e.*

$$\mathbf{v}(\mathbf{r}) = -\nabla u(\mathbf{r}) \ ?$$

If so, what is $u(\mathbf{r})$? If not, why not?